Managing Disruptions to Critical Interdependent Infrastructures in the Context of the 2001 World Trade Center Attack

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Overview

Critical infrastructure systems provide services that are essential to both the economy and well-being of nations and their citizens. As documented in a recent report to the U.S. Congress (U.S. General Accounting Office, 2001), it is of vital importance that these services not be degraded, whether by willful acts such as terrorism or by natural or random events such as earthquakes,

design flaws, or human error. Yet infrastructure systems and the organizations that manage them are now recognized as components of highly coupled systems that increasingly rely on one another in order to deliver key services. In addition, as complex, interconnected systems, they are vulnerable to disruptive events that propagate from system to system.

The September 11, 2001, attack on the World Trade Center in New York City illustrates the importance of understanding relationships among infrastructure systems and of managing these relationships in order to ensure continuance of necessary services after disruptive events. This research is intended to improve understanding of and support for the management of critical infrastructure interdependencies after large-scale, disruptive disasters. As discussed more fully below, infrastructure interdependencies occur when, due to either geographical proximity or shared operations, an impact on one infrastructure system is also an impact on one or more other infrastructure systems. The particular focus of this work is on developing techniques that can be used either to mitigate against or respond to events that have the capability of impacting interdependent infrastructure systems. The approach taken is to model salient elements of interdependent critical infrastructure systems and to provide decision makers with means of manipulating this model for purposes of mitigation or response. Models can provide powerful means of understanding (Wallace, 1994), monitoring, and controlling large-scale infrastructure systems (Beroggi and Wallace, 1998). The need for powerful but parsimonious models is particularly acute as modeled infrastructures increase in complexity, as when a number of infrastructures are interdependent.

The first objective of the present research—improving understanding—involves identifying, classifying, and describing incidents of critical infrastructure interdependence related to the World Trade Center attack. Three steps involved in accomplishing this objective are discussed. First, a coding methodology is presented for identifying instances of infrastructure disruptions, particularly related to interdependence that became evident after the World Trade Center attack. Second, results of the application of the methodology to newspaper articles for the period September 12, 2001, to December 12, 2001, are presented. Third, a number of potentially rich cases identified in step two and involving electric power and telecommunications infrastructures were developed and used in formulating and assessing the mathematical models. One such case serves as the basis for an exercise in the application of a mathematical model developed to contribute to the second objective, described next.

The second objective—improving support for the management of infrastructure interdependencies—involves development of analytic techniques embedded in computer-based tools. Such computer-based decision

support is intended to assist decision makers in reducing expected loss of service due to disruption and in restoring service more quickly if loss of service actually occurs. Definitions of infrastructure interdependence and related concepts are refined in order to allow development of a mathematical representation of infrastructure systems and their interdependencies. Next, such a mathematical representation is presented. The representation permits the development and use of algorithms for searching and locating solutions to problems associated with disruptions to interdependent critical infrastructures. A model of interdependent infrastructure systems operating under normal operating conditions is next developed. The model is intended for use in determining whether services provided by impacted infrastructures can be continued without undertaking extensive restoration operations. A second model is then presented to support decision making when the restoration of services is required. An example based upon restoration activities following the World Trade Center attack illustrates the application of this second model.

This paper proceeds as follows. The section on Strategies for Emergency Management places the problem of management of impacted critical infrastructures in the context of emergency response. The next section, Incident Identification and Classification, summarizes newspaper reports on post-World Trade Center impacts to critical infrastructure. Formal descriptions of infrastructure systems and their interdependency relationships are given in the section after that (Modeling Infrastructure Systems), followed by a discussion of how a network flow approach may be taken in modeling interdependent infrastructures. Models for supporting decision makers during response and restoration activities are presented in the section after that, and with their mathematical representations presented in the two appendices. A demonstration of the models' use is presented in the section entitled Model Demonstration. The paper concludes with a discussion of ongoing research and suggestions for further work.

Strategies for Emergency Management

The focus of the current research is on assisting emergency managers in responding to degradations in service that arise following events that impact infrastructure interdependencies. In these situations, infrastructure and emergency managers are faced with identifying, assessing, and mitigating the effects of the impact in order to restore necessary services. The implementation of strategies to achieve these goals requires responding personnel to marshal and apply available resources in a timely manner. Table 1 provides a construct that has been found useful in emergency management (Wallace, 1990) for discussing and analyzing a range of emergency management strategies. Mitigation and preparedness strategies are designed to reduce the impact from threats *before* disaster occurs, either by

Table 1. Emergency management strategies.

Strategy	Examples	When is Impact Reduced?
Mitigation	Building codes, insurance programs	Before occurrence, reducing the consequences, partially or in total
Preparedness	Warning systems, inventories of food and medical supplies	Before occurrence
Response	Rescue teams, fire fighting	After occurrence, as impact is being felt
Recovery	Disaster relief funds, rebuilding assistance	After occurrence, when full impact has been felt

reducing the impact of the disruption caused by the threat or by providing advance warning in order to lessen an event's impact. Response and recovery strategies are, on the other hand, designed to reduce the impact from threats *after* disaster occurs. As an example, response teams attempt to reduce impact by containing the effects of disruptions; disbursement of disaster relief funds is intended to lessen the burden on affected individuals and organizations.

Elimination or reduction of threats, whether they are human or technological, random or willful, is a key consideration in managing critical infrastructures. Assuming that for some threats it is possible to reduce, but not eliminate their probability of occurrence, additional management strategies are required to deal with such consequences as loss or degradation in service. The present research addresses pre-event strategies of mitigation and preparedness by increasing understanding of how organizations respond to disruptive events and by designing model-based tools for supporting organizational response to these events. Post-event response activities can be supported by identifying feasible alternatives for providing service and by assisting in developing new approaches to service restoration.

Incident Identification and Classification

Identification of instances of infrastructure dependence and interdependence is intended to support model-building (as described in subsequent sections) and expansion of knowledge of how organizations respond to infrastructure

disruptions. Instances of disruption to critical infrastructures in the borough of Manhattan are here summarized by drawing upon reports published in the *New York Times* Metro edition for the period September 12, 2001, to December 12, 2001. This time period closely approximates the length of the response phase to the World Trade Center attack.

Independent coders were provided with hard copy of all the above issues of the *New York Times*, along with definitions of critical infrastructures and with instructions on how to identify interdependency relationships among them. Eight infrastructures, as defined in the President's Commission on Critical Infrastructure Protection (1997), are included in the analysis: emergency services; transportation; information and communications; electric power; banking and finance; gas and oil production, storage, and transportation; water supply systems; and government. Additional definitions given to the coders were as follows:

An infrastructure is coded as **dependent** on one or more other infrastructures if any one of the following three conditions holds:

- **input**: the infrastructure requires as input one or more services from another infrastructure in order to provide some other service;

- **shared**: some physical components and/or activities of the infrastructure used in providing the service are shared with one or more other infrastructures;

- **exclusive-or**: either the infrastructure or some other infrastructure (but not both) can be in use during provision of the service.

Two or more infrastructures are coded as interdependent[1] if the following condition is true:

- **interdependent**: two or more infrastructures' physical components or activities are co-located within a prescribed geographical region.

Note that a disrupted infrastructure might not be involved in a dependent or interdependent relationship with another infrastructure. Thus, the final category:

- **none**: indicates that the infrastructure was not involved in a dependent or interdependent relationship with another infrastructure.

Results of the coding are summarized in Tables 2 and 3. Table 2 shows the number of disruptions for each of the eight infrastructure systems, regardless of whether or not a particular system was dependent or interdependent on one or more other systems. A total of 244 disruptions was

Table 2. Reported disruptions to critical infrastructure systems.

Infrastructure	Count
Emergency services	26
Transportation	44
Information and communications	29
Government services	43
Electric power	15
Oil and gas production and storage	2
Banking and finance	66
Water supply	19
Total	*244*

Table 3. Interdependence and dependence relationships.

Relationship	Count	Median
Input	18	2
Shared	1	1
Exclusive-or	2	2
Interdependent (co-located)	30	2
None	155	1

reported during the 90-day period. As shown in Table 3, 51 instances of interdependence or dependence were reported. The median number of infrastructure systems involved in a particular type of relationship is also given.

Based on these data, a number of potentially rich cases were identified. A number of organizations involved in these cases were contacted and asked to participate in a study to investigate interorganizational aspects of the management of these disruptions. Representatives of two public service providers agreed to be interviewed for this research to assist in choosing and developing a number of cases.

Modeling Infrastructure Systems

Definitions

Previous work provides definitions of the concept of critical infrastructure interdependence (President's Commission on Critical Infrastructure Protection, 1997; Little, 2002; Rinaldi et al., 2001), as well as discussions of key terms. Further refinement and generalization of these definitions are undertaken here before proceeding with the development of a mathematical representation.

An **infrastructure** is defined as a linked set of physical components with associated activities. **Physical components** are the built part of an infrastructure; **activities** are tasks necessary to operate physical components of the infrastructure. An **intersection** is the area where two or more physical components meet or are joined. An intersection circumscribes the activities and physical components necessary to manage the connection between the joined physical components. As an example, the intersection of two roadways may have one or more physical components (e.g., a traffic signal) and activities (e.g., manipulation of the signal via sensors embedded in the roadway). All intersections in a given infrastructure must have a physical component.

A **service** is something made available by the infrastructure for use or consumption. A service may be used by people or by other infrastructures; it is provided in order to meet a real or perceived need. An infrastructure can provide one or more services. **Material** is any physical entity or "substance or substances out of which a thing is or can be made" (Pickett, 2000, p. 837). Examples include electrons, people, product, and electromagnetic signals. Provision of a service requires activities such as movement, collection, transformation, or storage of material. Activities may be initiated at one or many locations and may be terminated at one or many locations. Assuming that traversal of a connection between two intersections requires a set of activities from beginning to end, management activities are necessary when provision of the service requires traversal of more than one intersection.

A **disruption** in an infrastructure is said to occur when one or more of the physical components or one or more of the activities needed to operate a physical component cannot function at prescribed levels. Disruption may or may not result in service degradation. **Service degradation** is said to occur when the service itself cannot be provided at its prescribed level.

An infrastructure is said to be **dependent** on one or more other infrastructures if any one of the following three conditions holds:

- **Input**: the infrastructure requires as input one or more services from another infrastructure in order to provide some other service.

- **Shared**: some physical components and/or activities of the infrastructure used in providing the service are shared.

- **Exclusive-or**: only one of two or more services can be provided by an infrastructure. (Note that a disturbance in an infrastructure that is dependent on another by virtue of its inability to operate if the other infrastructure is operating will affect just its own provision of service.)

A collection of infrastructures (denoted *I*) is said to be **mutually dependent** if the following condition holds:

- **Mutually dependent**: at least one of the operations of any infrastructure in *I* is dependent upon each of the other infrastructures in *I*. An example of mutual dependence involving two infrastructures occurs when an output of infrastructure A is an input to infrastructure B, and an output of infrastructure B is an input to infrastructure A.

Two or more infrastructures are said to be **co-located** if the following condition holds:

- **Co-located**: any of their physical components or activities are situated within a prescribed geographical region.

Collectively, these five conditions—input, shared, exclusive-or, mutual dependence, and co-location—will be denoted types of **interdependence**, since all imply that an impact on one infrastructure system is also an impact on one or more other infrastructure systems.

Examples of Infrastructure Interdependence

The following examples are intended to illustrate the above concepts of infrastructure interdependence.

Input Dependence
At 10:20 a.m. on September 11, 2001, after the collapse of the first World Trade Center tower, transit authorities decided to suspend all subway service, issuing an order to send all trains to their yards or to secure them in the tunnels. Around 10:15 several subway lines were left without alternating current, which supplies power to the trains through the third rails, and without direct current, which runs the signals. Officials did not know exactly how the power was disrupted. But because of the power loss, the closing of all stations in lower Manhattan and the possibility of further explosions or collapses, "the general consensus was that the best thing to do was discharge all passengers

and secure the trains temporarily," a transit authority spokesperson said (Kennedy, 2001, p. A8).

Shared Dependence

New York Waterway put all 24 of its boats into service, some to work as floating ambulances from piers in lower Manhattan and others to go to Hoboken, Hunts Point in Queens, and the Brooklyn Army Terminal (Kennedy, 2001).

Exclusive-or Dependence

After the collapse of the World Trade Center towers, financial services could not be provided because employees could not use the streets and sidewalks to travel to work. While there was pressure on financial firms to open, the firms worried that a partial opening might further damage investor confidence. They therefore pressed for a return to full operations. Some areas in New York's Financial District are narrow and congested even under normal circumstances; debris and vehicle and human traffic made them more so after the collapse. Moving people through lower Manhattan presented a challenge to the transit system (Berenson, 2001).

Mutual Dependence

This type of interdependency was not included in the review of *New York Times* reports. This type of interdependency would be said to occur when a power plant uses coal that is shipped by trains that require power from the plant in order to operate.

Co-location

There were numerous examples of both power lines and fiber optic cables being located in the same manhole, thus creating the possibility that the organizations responsible for these infrastructures would have to coordinate efforts at the manholes. A second example occurred at bridge and tunnel entrances which also served as the locations for security checks (*New York Times* Editorial Staff, 2001).

Modeling Infrastructure Systems as Networks with Flows

Infrastructure systems involve material flow, signals, water, commodities, people, and the like. A highway system, a power grid, a telephone network, or an airline network all provide the physical structure and associated activities to support the movement of material through the system (Berge, 1962).

A common feature in models of infrastructure systems is their essential dependence on a geometrical figure called a graph, a figure in a two-dimensional plane consisting of lines and points. In modeling the physical

components of infrastructure systems, both lines and points represent components of the system. Points are also called nodes or vertices, while lines are referred to as arcs, links, branches, or edges. A network is a graph where a direction is specified for every line, meaning that a line begins at one point and ends at another. In addition, lines in networks typically represent the movement of some material, whereas lines in graphs represent connections with the possibility of a direction. The physical structure of an infrastructure system can be represented by a graph; however, when the intention is to model an infrastructure providing a service, the graph must represent not only the activities necessary to generate, transfer, and terminate a service but also the service itself. Such a graph representation becomes a network with flows (Frank and Frisch, 1971), with points as nodes and lines as arcs, all with specified values representing characteristics of the infrastructure system being modeled. Physical components of an infrastructure are then modeled as a network, with nodes representing such components as communities, highway intersections, railroad yards, power generators, phone switching systems, and water reservoirs. In general, these are points where flow originates, is relayed, or terminates. The arcs of a graph can represent such elements as roads, railroad tracks, transmission lines, airline routes, and water pipes. In general, these are the channels through which material moves.

While the graph representations of different infrastructure systems may appear similar, the characterization of the nodes and arcs may be quite different. Telephone networks are characterized by parameters such as cost per unit length, capacity of a wire, and number of wires; a power grid will be characterized by parameters such as resistance, or capacitance. In addition, parameters that represent limitations and capabilities of nodes—sources of material, transshipment points, and destinations, and channels for material flow—can be incorporated into the graph model as numbers on the nodes and arcs. For power systems, a node representing power generation might have values for maximum power output, reliability of a generator, and cost per kilowatt-hour. An arc might have values representing capacity, reliability, and cost.

A network representation is very useful for modeling systems that involve connectivity. Indeed, in the models discussed below, the services provided by infrastructures are modeled as network flows. Given a system and its network, the question of determining if a particular material can be moved from one location to another can be understood as determining if there exists a path between two nodes. Since infrastructure interdependencies are connections, and the objective is to see if, after a disruption, material can be moved from a source to a particular destination (i.e., can be used to provide a service), a network representation is appropriate. In addition, to determine if a given level of service can be provided to a particular destination from a particular source, material flow over an arc can be represented by values on the nodes

and arcs. The objective here is to find the maximum flow between the specified locations and determine if it satisfies the desired level of service.

Interdependent infrastructures are here viewed as networks, with material corresponding to flows and with services corresponding to a desired level of these flows. For ease of representation, each network (i.e., each infrastructure) is defined as a collection of nodes and arcs with commodities (i.e., material) flowing from node to node along paths in the network. Activities, physical components, and intersections are considered to be contained within a node. Similarly, management activities are not considered in traversal of an arc; they are contained within the arc itself. For each commodity, each node will either be a supply node, a demand node, or a transshipment node. Arcs may, of course, have limited capacities (Ahuja et al., 1993). Infrastructure systems operate in an environment subject to disruptions—natural, human-caused, or willful acts. Given a system having interdependent infrastructures, the analysis must determine likely system degradation following an event. Based upon performance criteria, an infrastructure system can then be designed to minimize possible service degradation. In addition, once a disruption occurs, alternative ways of restoring service can be determined.
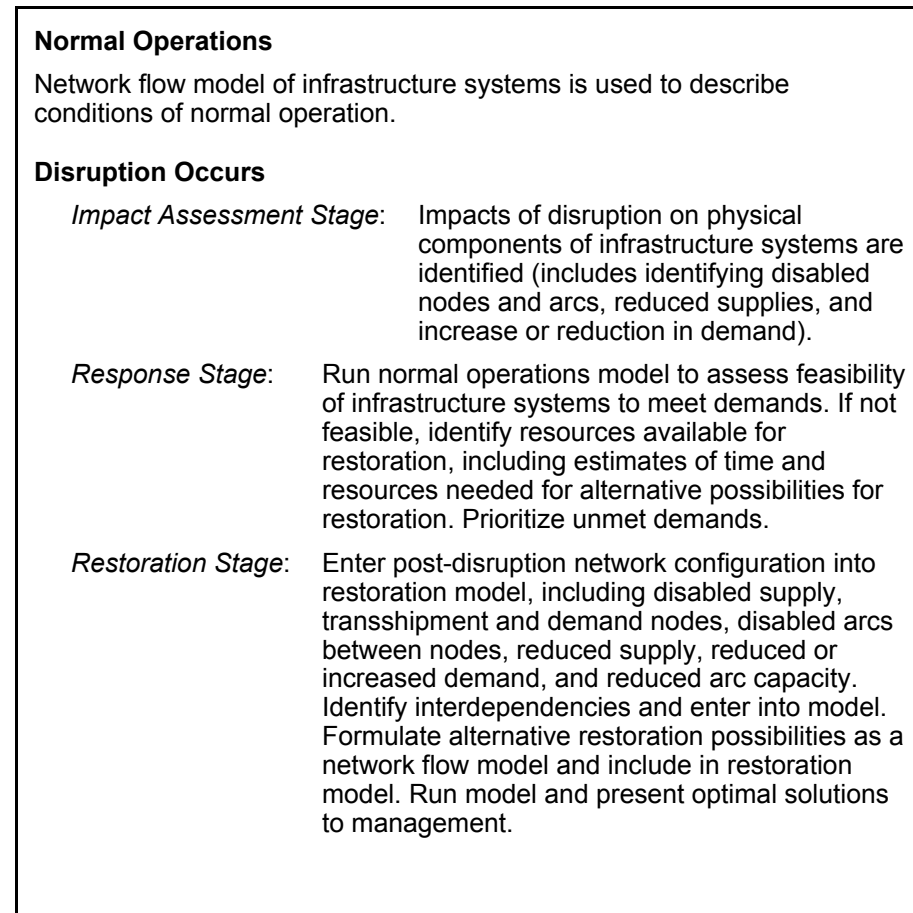
The following section employs a network with flows representation in the formulation of models to aid emergency managers in response and, if necessary, restoration of service.

Decision Support for Response and Restoration

The focus of the present research is on assisting organizations responsible for responding to events that disrupt services provided by infrastructures they manage. Managers in these organizations are responsible for developing response and restoration strategies and proposing them for review by stakeholders or regulators both within and external to their organization. Once a strategy has been determined, it is implemented by field personnel. This section describes models intended to be embedded in computer-based systems to support strategy development, both before and after the occurrence of disruptive events. In the context of management of interdependent infrastructures, these strategies are likely to be directed towards restoration of key services. Figure 1 presents stages of decision making for emergency response and restoration actions using the models.

Normal Operations

Under normal conditions, all the demands of all the infrastructures are met. Since interdependency considerations come into consideration only when there are unmet demands, each infrastructure may be considered to be operating independently and analyzed independently. It is assumed that, before the disruption, the system operates at a minimum cost-optimal

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Normal Operations

Network flow model of infrastructure systems is used to describe conditions of normal operation.

Disruption Occurs

Impact Assessment Stage: Impacts of disruption on physical components of infrastructure systems are identified (includes identifying disabled nodes and arcs, reduced supplies, and increase or reduction in demand).

Response Stage: Run normal operations model to assess feasibility of infrastructure systems to meet demands. If not feasible, identify resources available for restoration, including estimates of time and resources needed for alternative possibilities for restoration. Prioritize unmet demands.

Restoration Stage: Enter post-disruption network configuration into restoration model, including disabled supply, transshipment and demand nodes, disabled arcs between nodes, reduced supply, reduced or increased demand, and reduced arc capacity. Identify interdependencies and enter into model. Formulate alternative restoration possibilities as a network flow model and include in restoration model. Run model and present optimal solutions to management.

Figure 1. Process of decision support for response and restoration.

solution, denoted **normal operations**. This solution can be found by solving the normal **operations model** for each infrastructure system separately. This network flow model is described in detail and represented mathematically in Appendix A.

The normal operations model consists of an objective to be optimized and constraint requirements representing flow conservation and structural requirements. The objective is to minimize the cost of operation of each of the infrastructure networks, while satisfying demand. Therefore, constraining the solution is the requirement that the flow out of the supply nodes must be less than the available supply and the flow into the demand nodes must meet the

required demand. Since transshipment nodes have neither supply nor demand, the flow into a transshipment node must equal the flow out. Also, we cannot exceed the capacity of transshipment nodes. Similarly, the flow on any arc cannot exceed its respective capacity. Last, the flow on the arcs must be nonnegative, with a value of zero denoting that there is no flow on that arc. Structural requirements model the network's configuration and identify whether or not arcs and nodes are in operation.

The normal operations model is intended for use by infrastructure managers during routine operations. However, it is envisioned that the model is also a component of a decision support system to be used by a state or local emergency operations center for emergency response and restoration.

Impact Assessment Stage

When an incident occurs that has the potential to cause a major disruption in service, initial activities include assessing (1) its likely impact on physical components of infrastructure systems, (2) the potential loss of service, (3) its impact on the safety of humans, and (4) its effect on the security of sensitive systems in the natural and built environments. Reductions either in capabilities of supply and transshipment nodes or capacities of the arcs between the nodes need to be identified. Assessment of new demands must also be made, since post-event conditions can result not only in decreases but also in increases to demand.

Response Stage

The impact assessment may reveal that the desired or normal demand levels cannot be met. However, once the desired demand levels are ascertained and prioritized, it may be possible to satisfy revised demands using functioning supply points that, before the event, had been operating at less than full capacity. Absent such a situation, the normal operations model must be modified by changing supply, transshipment, and demand nodes and flow conservation constraints. In addition, connections between nodes must be revised—either eliminating arcs or reducing capacity over an arc—to reflect damage caused by the incident, i.e., changing the structural constraints. The model is then run, taking into account the revised and prioritized demands, to determine the set of feasible solutions. If at least one feasible solution is found for each infrastructure system, response to the incident can proceed. If not, alternatives for restoring service must be developed. The infeasible solution provided by the modified normal operations model for the infrastructures that could not provide a service, i.e., meet the revised demands, may be used to identify unmet demands.

Restoration Stage

Once the need for developing ways of restoring service has been determined, physical and personnel resources available for implementing restoration strategies must be identified. Time may be one of the factors considered in selecting a restoration strategy and may indeed be critical. Many infrastructure organizations have database systems with visualization capabilities that enable rapid determination of resource availability and location. A **restoration model** for decision support in selecting restoration strategies is described in this section; Appendix B provides more detail, including the mathematical representation.

As previously noted, if all the revised demands for all the infrastructure services can be met without restoration, each infrastructure system can be considered to be operating independently. However, when unmet demand for any infrastructure service is found, interdependencies among infrastructure systems must be considered and incorporated in any model that seeks to support the restoration decision-making process. Also, this process must involve those emergency managers who have the authority to make decisions concerning prioritization of unmet demand among systems.

The material in the remainder of this section will describe how the various interdependencies defined above are represented in the restoration model. Each interdependency will be described with a textual description of how it is modeled. Then the overall model will be discussed.

Input

Input dependency is modeled as follows. Consider a telephone switching station. The switching station itself is a transshipment node within the telecommunications network. However, this same switching station from the perspective of the electrical network is seen as a demand node since it needs an adequate source of electricity to operate. From the perspective of the electrical network, the switching station is therefore a dependent component. More formally, denote the demand node for the switching station in the electrical network to be node j. If there is an adequate flow of electric power into node j, the switching station can function. If power is not available at this level, then the switching station fails. A binary variable, y, is used in this case to represent the two states of the switching station. If adequate power is available at j, then $y = 1$; if not, then $y = 0$. The phone switching station also has some maximum capacity u within the telecommunications network. Consider the station's capacity to be the product of the binary variable y and the rated capacity w. When adequate power is available the station can operate to its capacity w (since $y = 1$). On the other hand, if adequate power is not available then the capacity of the station is 0. This binary variable y serves as a virtual connector between the two systems. Its value is set by the conditions

existing in one system, and affects the operating characteristics of a second system. Events affecting the power network that have an effect on node j in turn impact the model of the telecommunications network. The effect on each system can be analyzed in a similar manner.

Shared

Methods similar to those used in multi-commodity flow problems (Rardin, 1998) may be used to model systems with the shared dependency. The use of one or more shared components by all systems is constrained by a limit on maximum flow. In the context of the example of ferry service given earlier, regardless of whether or not the ferry is used for transit services or medical services, it cannot exceed its maximum capacity. In the context of the telecommunications and power systems and the World Trade Center restoration, it could have been advantageous to route the shunts used to restore phone and power through the same temporary enclosures. This would reduce time and cost since only one enclosure would need to be built.

Exclusive-or

When multiple infrastructures share a component, for example, an arc, but the component can only be used by one infrastructure at a time, exclusive-or dependence occurs. In the example given previously, streets (i.e., shared components) could not be used by both the emergency response personnel and financial district workers. Considering power and telecommunications, it can also be the case that a power and a telecommunications shunt cannot be routed in close proximity to each other. This would be the case with a T1 line, which cannot be too close to a power line due to radio frequency interference considerations. Emergency managers would establish the priority for which lines could be run, i.e., which services are more critical, and others desiring a similar path would have to be re-routed. Exclusive-or dependencies are modeled by determining which service has the highest priority, making an appropriate change to the objective function, and selecting additional constraints to restrict flow to one commodity or the other.

Mutual Dependence

A collection of infrastructures is said to be mutually dependent if at least one of the operations of any infrastructure system is dependent upon any other infrastructure system and at least one of the operations of this other infrastructure system is dependent upon the first infrastructure system. As previously noted, no cases of mutual dependence in infrastructure systems were identified in news reports from the *New York Times*. However, consider two mutually dependent systems, a natural gas system pump and a gas-fired electric power generator. From the perspective of the natural gas system, the pump is a transshipment node and the generator is a demand node. From the

perspective of the electrical network, the generator is a supply node and the pump is a demand node. The generator needs gas to produce electricity; the pump needs electric power to deliver gas through the system to the generator.

In this case, two variables are used: y_{gas}^{power}, the connection of the electricity to the pump, and y_{power}^{gas}, the connection of the gas to the generator. Failure of one component causes its corresponding binary variable to be set to zero, thus reducing the capacity of the other component to zero. In other words, if the pump were to fail, supply of gas to the generator would be inadequate and then y_{power}^{gas} would be set to zero. When $y_{power}^{gas} = 0$, the capacity of the generator is now zero (since its capacity is the product of y_{power}^{gas} and its capacity u). Since the generator is a supply node, all flows on the arcs (i.e., the power lines) leaving the generator would now be zero, by flow conservation.

Alternately, if there is a lack of power at the pump demand node in the electrical generating network, its binary variable y_{gas}^{power} is set to zero and the capacity of the pump reduced to zero. To correct this situation, either an alternate source of gas must be found for the generator or an alternate source of power must be found for the pump.

Co-location

Co-location occurs when activities or physical components of two or more infrastructures are situated within a prescribed geographical region. An event that impacts the entire region, as occurred in the World Trade Center attack, impacts all infrastructures in that region. The implication for any restoration model is that capacities of the supply, demand, and flow capacity nodes and arcs in the model must be revised based on their location with respect to the impact of the disruptions. This determination is made during the assessment stage of the decision process. Once necessary adjustments have been made, the normal operations model (see above) can be run to determine any feasible paths through the networks. If no feasible paths exist, the restoration model discussed in the remainder of this section is run to assist in determining where alternative nodes and arcs can be constructed.

Summary of Restoration Stage

In the restoration phase, alternatives for restoring services are considered. The objective is to find the alternative that meets unmet demand at a minimum cost. However, different demands for the same service as well as demands for different services from the same source will likely emerge and must therefore be reconciled and prioritized. The objective function of the restoration model must be able to incorporate different priorities in addition to modeling interdependencies.

Once all the interdependencies have been identified, two sets of demand nodes can be defined among all the infrastructure systems. One set would be those nodes that do not affect nodes or arcs in any other infrastructure. We will call these nodes independent. The remaining demand nodes would be associated with a connection to some other infrastructure. These would be the dependent nodes.

In the restoration model, we must also meet the flow conservation constraints. That is, the flow from supply nodes, through transshipment nodes, and to demand nodes must balance to satisfy the demand while not exceeding the supply. The node capacity constraint must be modified to represent the fact that the total flow into the node would be less than or equal to its revised capacity, w, multiplied by the connector variable, y. Constraints are included in this restoration model to shift the connector variable from 1 (operating) to 0 (failed) when the required demand is not met at a dependent node. As in the normal operations model, arc flow is limited to its capacities. Supply and demand constraints as well as structural constraints representing the reconfigured network are also in the model.

Note that some interdependent infrastructure system failures may result in reducing capacity to some value other than zero. For example, loss of supervisory control systems in a subway system may result in operators' exercising greater care and slowing trains, so that the post-disruption capacity may be one-third of normal. In this case, the connector variable y would shift between 1 and 1/3. The exact effect of each disruption must be evaluated.

In summary, the restoration stage consists of first identifying each of the interdependencies among infrastructure systems, modeling them, and incorporating them into the restoration model. Emergency managers must revise and prioritize the supplies and demands in order to have the restoration model provide support to those responsible for restoring services.

Model Demonstration

The following excerpts from a news report on September 14, 2001 (Pristin, 2001, p. A12), illustrate impacts of the World Trade Center attack on electrical and telecommunications infrastructures. The article states that "[i]n Lower Manhattan, about 300 Con Edison workers are trying to restore service to about 12,000 commercial and residential buildings without electricity," according to a spokesman for the utility. The previous day, electrical, gas and steam service "were normal throughout the city, except for the areas affected by the loss of two substations that were knocked out when 7 World Trade Center caught fire and collapsed." Of the approximately 500,000 phone lines south of 14th Street, at least 200,000 remained out of service on September 14 (though "most of those lines served locations that are either not in use or no longer exist").

Five Verizon switching centers—one of which is on West Street near the location of 7 World Trade Center—serve these 500,000 lines. The loss of power to the West Street switching center affected the 200,000 phone lines below 14th Street and also about three million private data lines for corporate customers. About 20% of these data lines that serve the New York Stock Exchange were among those affected. "Even more than the West Street office," the same article notes, "the New York Stock Exchange depends on a Verizon switching center on Broad Street that handles about 80 percent of the exchange's data lines." The center on Broad Street was not physically damaged by blasts from the attack but did lose power shortly afterward. As of September 14, diesel generators were continuing to provide power, and power had been restored to the site on September 12.

The foregoing news report illustrates both input and mutual dependence and serves as the basis for the example used to exercise the modeling tools presented earlier in this section. Data associated with the World Trade Center attack (e.g., locations of equipment and personnel, generating capabilities, capacities of feeder lines and shunts, power demands) is, of course, sensitive and has not been used. Rather, the example takes the incident as a starting point and is supplemented with additional, simulated events (e.g., an additional instance of mutual dependence, a system failure) and with information on aspects of operations provided by Consolidated Edison and Verizon personnel and by other domain experts.

To illustrate the mutual dependence of telecommunications and power, damage to a Controlled Environment Vault (CEV) in the phone system is simulated. The result is a loss of phone service. With this failure in the phone system, the supervisory control and data acquisition (SCADA) system for the power company becomes unable to notify supervisors of the impending failure of another component, which therefore also fails. Figure 2 includes a depiction of a section of these systems after impact assessment. (In Figure 2, CO refers to the Verizon Central Offices where cables from the CEVs enter and switching of calls occurs.) The power system considers only the distribution portion. The distribution system starts at the high voltage supply feed to a 13kv substation. The lines from the feeder to the 120/208v transformers and the demand nodes will be the customers.

Illustrative Example

Referring to Figure 2, the demonstration example begins with the failure of Transformer *A*, which is in the Pearl Street feeder. This failure results in loss of power to Metropolitan Transit Authority (MTA) facilities and systems and to one CEV and residential and commercial customers. The failed CEV had been carrying SCADA lines in addition to residential and business service. This SCADA system had been monitoring the power feeder and associated
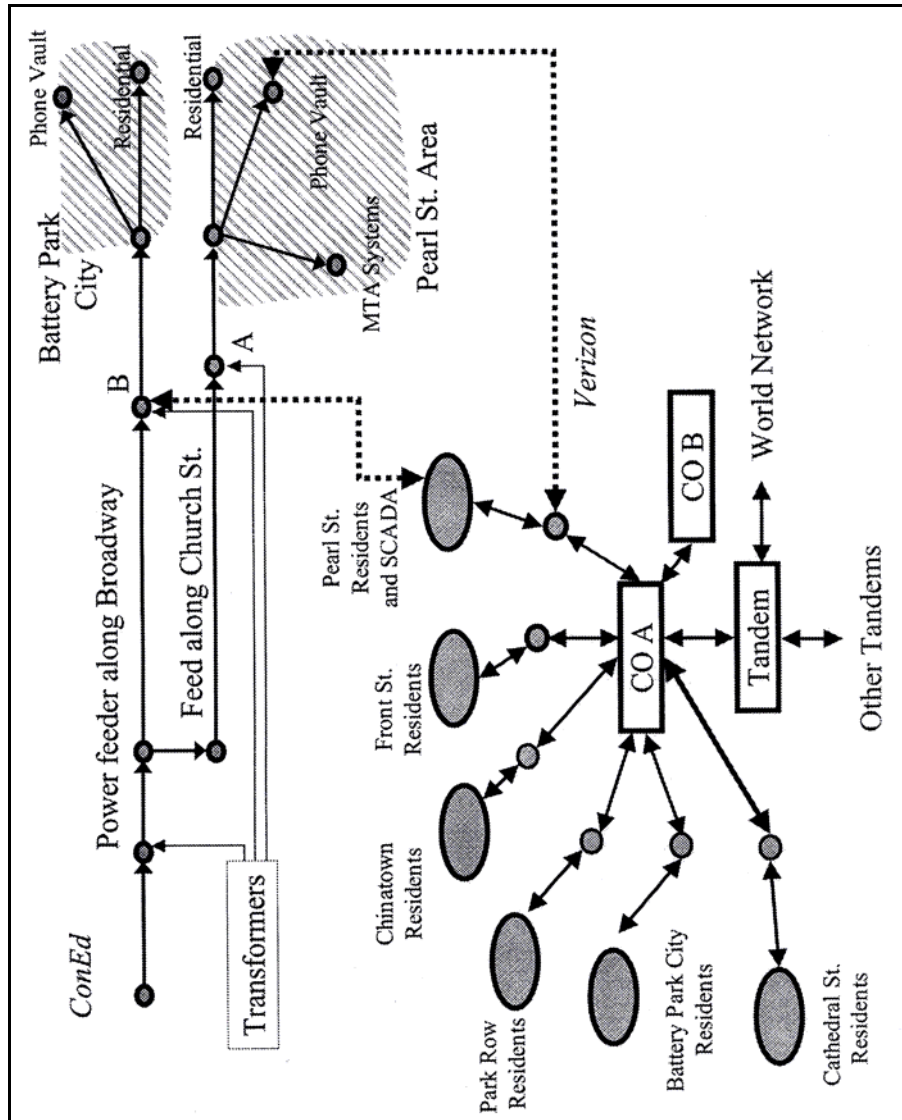
**Figure 2. Interdependent telecommunications and power infrastructures.**

components along Broadway. Consequently, failure of the SCADA results in an inability to monitor these power distribution components. A high temperature condition at Transformer *B* goes unreported and the transformer subsequently fails. This failure results in loss of power to another CEV and another residential/commercial area. Another CEV failure leads to loss of phone service in a second area.

After assessing the impact and making the required modifications, the normal operations model is run and determines that no feasible solutions exist for power and telecommunication. The model also identifies unmet demands. The next step in the decision process is to examine the solution generated by the model, identify unmet demands, and obtain priorities from managers on restoration. During the response stage, available resources are identified. If the capacity at the substations were to exceed the required load for the demands, the restoration strategy would be to develop a routing plan for feeder lines and inform managers of the demand level and the routing plan. However, if these substations do not have the power available to satisfy the demands, then restoration will require the use of temporary generators.

In this example, it is assumed that managers place priority on restoring power to the two CEVs (requiring five and six units each) and the MTA systems (requiring 10 units). Managers conduct a resource assessment and determine that the three closest substations to the affected areas each have five units of power capacity available. However, these 15 units are not sufficient for the three loads that are candidates for restoration. There are also two portable diesel generators, each having capacity of four units, which can be moved into the area. Managers identify four possible sites for the generators. Each site can accommodate up to two generators.

Figure 3 depicts the decision situation facing the emergency managers. A question is what combinations of transformers (substations) and temporary generators should be selected for presentation to stakeholders (e.g., New York City Office of Emergency Management). Temporary feeder lines or shunts have to be routed from a supply site (e.g., substation or generators), to a demand site that requires resources (e.g., cable, crews). In addition, one or more sites to house the generators must be selected, based on consideration both of the proximity of the generator site to the demand site, and of the resources required to relocate the generators. In this example, there are 33 possible ways to satisfy the unmet demand. Actual implementation, including routing through city streets, must be planned but is not considered here.

The restoration model is designed to support selection of a restoration strategy. The constraints have been specified in terms of available resources and unmet demand; the objective function that drives the search algorithm needs to be specified. Based on discussions with domain experts, reasonable cost estimates have been developed and are depicted in Table 4.
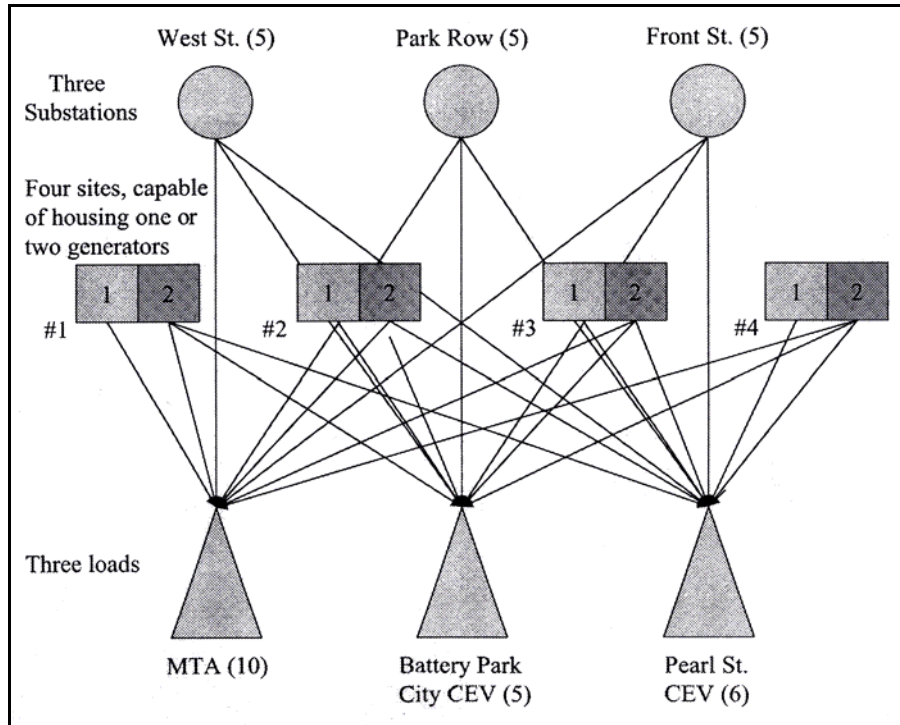
Figure 3. Decision situation.

Since the temporary generators could be located closer to the demand sites than the distance to the transformer substations, shunts from the generators could be installed at lower cost than those from substations. However, transportation and installation of generators do incur costs. With these values, the restoration model can again be run to provide the best alternative restoration strategies, as discussed next.

The decision situation facing the emergency managers is, in essence, to construct a new network utilizing power from the substations from unaffected sections of the power grid. This restoration model does not require the y variables in its formulation. The specific objective function for this example is to minimize cost of operation of the shunts and the generators. The restoration model is formulated as follows. Each shunt has fixed cost, k, and power cost, c, which is a function of generator or transformer use at the substations. A unique k is determined for each shunt and includes the cost of generator transport and setup as appropriate. The cost of power is set at 1 for power

Table 4. Estimated generator and shunt costs.

Site	Location, Qty.	Demand Sites (shunt cost + generator cost)		
		Metropolitan Transit Auth.	Battery Park City CEV	Pearl Street CEV
1	substation, 1	30+0	40+0	50 +0
2	substation, 1	40+0	30+0	40 +0
3	substation, 1	50+0	40+0	30 +0
4	generator site 1, 1 gen.	25+10	35+10	55 +10
5	generator site 2, 1 gen.	35+10	25+10	45+10
6	generator site 3, 1 gen.	45+10	30+10	35+10
7	generator site 4, 1 gen.	55+10	40+10	25+10
8	generator site 1, 2 gen.	25+20	35+20	55+20
9	generator site 2, 2 gen.	35+20	25+20	45+20
10	generator site 3, 2 gen.	45+20	30+20	35+20
11	generator site 4, 2 gen.	55+20	40+20	25+20

coming from the distribution grid and to 1.5 for power from generators. The cost to operate each shunt is therefore $c_e^i\, x_e^i\, +\, k_e^i$. Using binary variables r to indicate whether or not a shunt is installed, set $r_e^i = 1$ when $x_e^i > 0$. The objective is to minimize the total cost of installing and operating all shunts, as follows:

$$\text{minimize} \quad \sum_{e \in E^i} c_e^i x_e^i + k_e^i r_e^i$$

$$\text{subject to} \quad \sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i \geq b_j^i \qquad \text{for } j \in V^i \quad \text{with } b_j^i < 0$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = b_j^i \qquad \text{for } j \in V^i \quad \text{with } b_j^i \geq 0$$

$$x_e^i \leq 100 r_e^i$$

$$x_e^i \geq 0$$

$$r_e^i \quad \text{binary.}$$

In order to meet the constraint of only two diesel generators, additional binary variables w and t are introduced. The variable t is assigned a value of one if there is flow from a one-diesel generator site. The variable w is assigned a

value of one if there is flow from a two-generator site. Therefore, there are four t variables corresponding to the four sites having one diesel generator each, and four w variables for the four sites if they have two diesel generators. The number of diesel sites is controlled by constraining the sum of the t variables to be less than or equal to two, and the sum of the w variables to less than or equal to one. Additionally, w is constrained to zero if t is greater than zero, and t is constrained to zero if w is greater than zero.

For this example, solution of the restoration model produces two solutions with minimum costs, as shown in Figures 4 and 5. Solution 1 has two parts: (1) to run shunts from the West Street and Park Row substations to the MTA facilities, and from the Front Street substation to the Battery Park City CEV and (2) to locate two diesel generators at site 4 to power the Pearl Street CEV. Solution 2 also has two parts: (1) to run shunts from the West Street and Front Street substations to the MTA facilities, and from Park Row substation to the Battery Park City CEV and (2) to locate two diesel generators at site 4 to power the Pearl Street CEV.

Given these solutions, managers can now decide which solution to pursue. However, if neither is found to be acceptable, it may be possible to incorporate other costs into the objective function and to develop another set of solutions for review. If a restoration strategy is proposed, the additional cost of using that strategy versus one of the minimum cost strategies can be determined. Note that estimates of these costs result from evaluation by managers of available resources and of the time and effort required to deploy these resources.

Conclusions and Suggestions for Further Research

The nature of the September 11th attack on the World Trade Center—its scale, scope, and type of target—demonstrates the need for better understanding of the interdependencies among critical infrastructure systems. This research employs a systems approach to addressing this need by modeling interdependent infrastructures as systems of systems. The approach allows for optimization of restoration strategies and is a step towards integration of models of infrastructure interdependence with decision support systems. In the future, models such as those presented here could be used to identify opportunities for reducing vulnerabilities, developing countermeasures to mitigate impact of disruptions, and guiding actions for response and recovery.

Summary of Work to Date

Initial investigations of infrastructure interdependence as reported in the *New York Times* produced a starting set of incidents, some of which have been and continue to be explored with the appropriate organizations. In order to
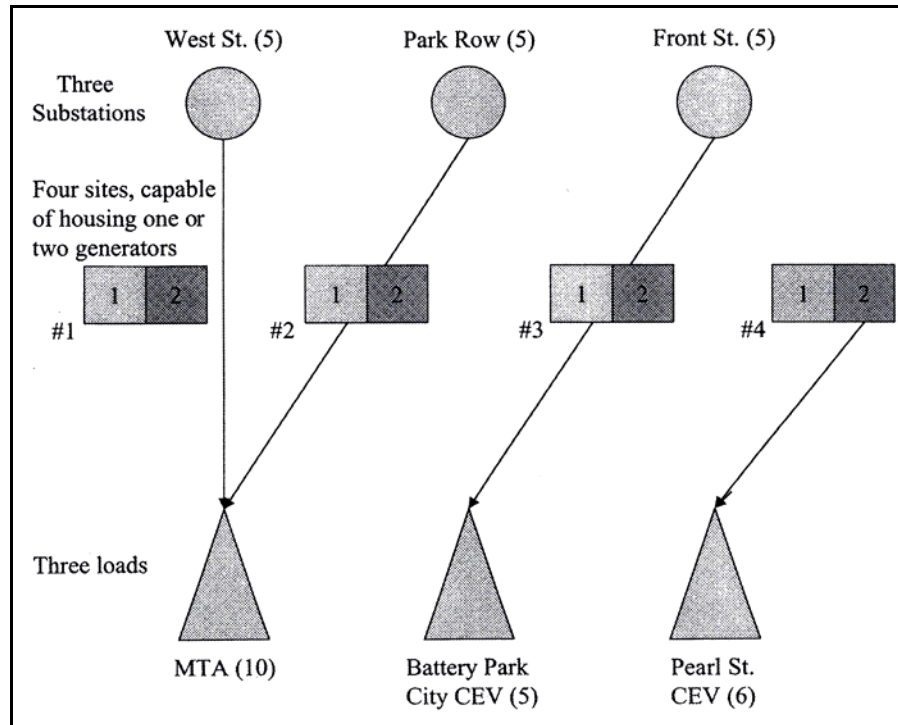
Figure 4. Solution 1 to decision situation.

conduct modeling in later stages of the work, definitions of interdependence among and between critical infrastructure systems were operationalized. A mathematical representation of the physical components of an infrastructure system and the services it provides was developed. A decision support process was proposed to assist infrastructure managers in responding to disruptive incidents that involve infrastructure interdependencies. Two models, one for more normal operations and one for restoration of services, were proposed as components of a decision support system. An illustrative example centered on events following the World Trade Center attack was presented to demonstrate how the models could be used in response and restoration.
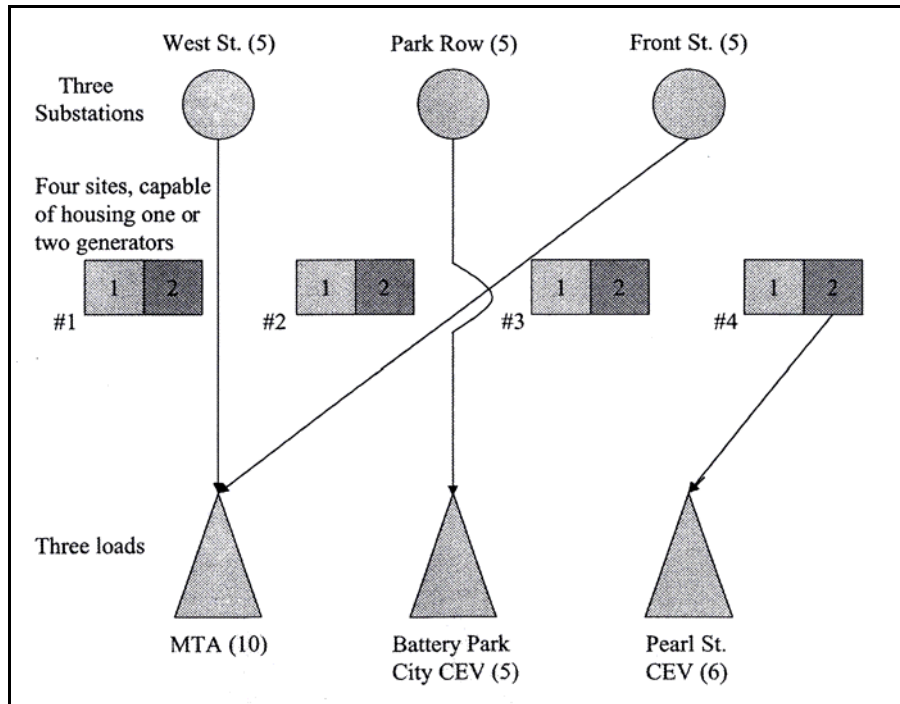
Figure 5. Solution 2 to decision situation.

Ongoing and Future Work

Ongoing work is being conducted on improving understanding of decision-making processes in the management of infrastructure interdependencies. Based on data from *New York Times* reports, a number of potentially rich cases of infrastructure dependence and interdependence have been identified. Organizations involved in these cases are participating in a study to investigate decision making in the management of these disruptions. Of particular interest are non-routine cases, since these provide an opportunity to examine organizational flexibility and improvisation (Kreps, 1991). Additionally, since one concern for many organizations is the proprietary nature of relevant data, cases in which ad hoc or temporary solutions were employed are also being investigated. The Critical Decision Method (Klein et al., 1989; Hoffman et al., 1998; Flanagan, 1954) of knowledge elicitation is well-suited to these situations. The Critical Decision Method (CDM) can be

used for uncovering information about how individuals responded to critical situations. It is intended to uncover critical decisions and their content, particularly for non-routine decision making. It has proven useful for guiding training, identifying lessons learned, and developing decision support tools (Klein et al., 1989). Additional information is being provided through an examination of other materials such as activity logs, maps, and after-action reports. Approximately five cases involving emergency services, electric power, and telecommunications are currently being investigated, with a number of participants being interviewed for each case. Results of these investigations will continue to inform the construction of models to support the management of infrastructure interdependencies.

Additional modeling efforts are focusing on addressing the various types of interdependence, again drawing upon case studies. Efforts are also underway to incorporate considerations of time into the models, since some effects of a disruption in service take time to develop. For example, a generator may be able to produce additional power to cover a shortfall only for a limited amount of time. These and other time-varying consequences of disruption should appear in the restoration model. The result is in a **time-expanded network** (Ahuja et al., 1993). Longer-term work may involve the use of models as an aid in exploring vulnerability of systems, particularly during planning and design of infrastructure systems. One approach is to develop event scenarios, evaluate network performance in the scenarios, and re-design as necessary.

Finally, it should be noted that visual models capitalize on a fundamental, native expertise of humans: the capability to solve complex problems by reasoning with graphical representations. Visual models can offer advantages over purely lexical models by increasing interpretability and reducing cognitive load, thus enabling decision makers to devote additional cognitive resources to problem solving (Larkin and Simon, 1987). Indeed, informal observation on the results of interviews conducted for this research suggests that visualization tools such as geographic information systems had widespread use during the response to the World Trade Center attack. Future work should contribute to capabilities for visualizing both the assumptions and implications of models of infrastructure interdependence.

Notes

1. As discussed below, the term "co-located" is later used for this definition. The term "interdependent" is later redefined.

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Acknowledgments

Appendix A

Normal Operations

As discussed in the section on Modeling Infrastructure Systems, interdependent infrastructures are modeled as networks having services represented as flows in networks. Let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Associated with each node $j \in V^i$ is a scalar b^i_j representing its supply or demand. If node $j \in V^i$ is a demand node then $b^i_j < 0$; if it is a supply point then $b^i_j > 0$; and if it is a transshipment node then $b^i_j = 0$. If $j \in V^i$ is a supply node then b^i_j equals the maximum possible amount that could be produced at that node. A non-negative vector of variables, x^i, represents the flow on each arc of the infrastructure. Associated with each arc in E^i are non-negative costs c^i and capacities u^i, where $0 \leq x^i \leq u^i$ for each element in x^i. The objective is to find the minimum cost feasible network flow under normal operating conditions.

Arcs are represented using either the endpoints of the arc or the index of the arc. For a node $l \in V^i$ for some infrastructure $i \in I$, let $\delta^+(l)$ denote the set of arcs in E^i that enter node l and let $\delta^-(l)$ denote the set of arcs in E^i that leave node l. Define $\delta(l) := \delta^+(l) \cup \delta^-(l)$, the set of all arcs incident to node l. Without loss of generality, assume that every supply

node has no incoming arcs (i.e., $\delta^+(l) = 0$ if $b_l^i > 0$) and that demand nodes have no outgoing arcs, (i.e., $\delta^-(l) = 0$ if $b_l^i < 0$). A transshipment node l may have a limited capacity, w_l^i, modeled by placing an upper bound on total flow across the arcs $\delta^+(l)$. Included in the model are flow conservation constraints (i) that for supply nodes ensure that total flow out of the node is no greater than the available supply, (ii) that for demand nodes ensure that demand is met, and (iii) that for transshipment nodes ensure that flow into the node equals flow out of the node.

Under normal conditions, all the demands of all the infrastructures are met. Since interdependency considerations come into consideration only when there are unmet demands, each infrastructure may be considered to be operating independently and analyzed independently. It is assumed that, prior to disruption, the system operates at a minimum cost feasible solution, denoted the normal operations solution. This solution can be found by solving the model for each infrastructure separately. The model prior to disruption is the solution to the following minimum cost network flow problem for each infrastructure $i \in I$, where the total flow into node j is given by $\sum_{e \in \delta^+(j)} x_e^i$ and the total flow out of the node is given by $\sum_{e \in \delta^-(j)} x_e^i$, as follows:

$$\text{minimize} \qquad \sum_{e \in E^i} c_e^i x_e^i$$

subject to $\quad \sum_{e \in \delta^-(j)} x_e^i \leq b_j^i \qquad$ for $j \in V^i$ with $b_j^i > 0$

$$\sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i < 0$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = 0 \qquad \text{for } j \in V^i \text{ with } b_j^i = 0$$

$$\sum_{e \in \delta^+(j)} x_e^i \leq w_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i = 0$$

$$x^i \leq u^i \qquad \text{for } i \in E^i$$

$$x^i \geq 0 \qquad \text{for } i \in E^i$$

Appendix B

Restoration Model

In the restoration phase, alternatives for restoring services are considered. Different demands for the same service as well as demands for different services from the same source will likely emerge and must therefore be reconciled and prioritized.

Define the set $V^{i,+} \subseteq V^i$ to be the nodes $j \in V^i$ with $b_j^i > 0$ (supply nodes); sets $V^{i,=} \subseteq V^i$ (transshipment nodes) and $V^{i,-} \subseteq V^i$ (demand nodes) are defined similarly. Let $D(i, i_1) \subseteq V^{i,-}$ be the set of nodes in i that some other infrastructure i_1 depends upon (parent nodes) and let $D^i := \cup_{i_1 \in I, i_1 \neq i} D(i, i_1)$. This subset of nodes is the interdependent nodes. The remaining nodes in $V^{i,-}$ will be referred to as the independent nodes. The binary variable $y_{i_1,j}^{i,l}$ is the connection between node l in infrastructure i (where it is a demand node) and node j in infrastructure i_1, where it may be either a supply, demand or transshipment node and is only defined for $l \in D(i, i_1)$ (this connector variable concept was discussed in section entitled Restoration Stage). Let $C(i_1, i) \subseteq V^{i_1}$ be the set of nodes in i_1 that depend on some other infrastructure i (child nodes) and let $C^{i_1} := \cup_{i \in I, i \neq i_1} C(i_1, i)$. Without loss of generality, all nodes have been disaggregated to the point where, given infrastructures i, i_1, and l in $D(i, i_1)$,

there is a unique node j in $C(i_1,i)$ such that $y_{\hat{n},j}^{i,l}$ is defined, and given infrastructures i, i_1, and node j in $C(i_1,i)$, there is a unique node l in $D(i,i_1)$, such that $y_{\hat{n},j}^{i,l}$ is defined.

The objective function of the restoration model must be able to incorporate different priorities in addition to modeling interdependencies. On independent nodes, the available supply may meet the required demand, but there may be some shortfall. The slack variable s_j^i represents the shortfall in meeting demands at independent nodes. In the model, there is no consideration for partial slack at the interdependent nodes. Because these interdependent nodes control the operation of nodes in other infrastructure systems, they either must be fully operational or they are in a failed condition. There is no benefit to partially meeting the requirement. After the response phase, when the operator realizes there are unmet demands across one or more systems, one choice for the objective function is to minimize the total shortfall (slack) plus the unmet interdependent demands, as follows:

$$\text{minimize} \quad \sum_{i \in I} \sum_{j \in V^{i,-} \setminus D^i} k_j^i s_j^i + \sum_{i \in I} \sum_{l \in D^i} \sum_{\hat{n} \neq i} b_l^i (1 - y_{\hat{n},j}^{i,l})$$

$$\text{subject to} \quad \sum_{e \in \delta^-(j)} x_e^i \leq b_j^i \qquad \forall j \in V^{i,+}, \forall i \in I$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = b_j^i \qquad \forall j \in V^{i,=}, \forall i \in I$$

$$s_j^i + \sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \forall j \in V^{i,-}, \forall i \in I$$

$$\sum_{e \in \delta^+(j)} x_e^i \le w_j^i \qquad \forall j \in V^{i,=}, \forall i \in I$$

$$\sum_{e \in \delta^-(j)} x_e^{\hat{n}} \le b_j^{\hat{n}} y_{\hat{n},j}^{i,l} \qquad \forall j \in C(\hat{n},i) \text{ with } b_j^{\hat{n}} > 0, \forall i, \hat{n} \in I, i \ne \hat{n}$$

$$s_j^{\hat{n}} + \sum_{e \in \delta^+(j)} x_e^{\hat{n}} = -b_j^{\hat{n}} y_{\hat{n},j}^{i,l} \qquad \forall j \in C(\hat{n},i) \text{ with } b_j^{\hat{n}} < 0, \forall i, \hat{n} \in I, i \ne \hat{n}$$

$$\sum_{e \in \delta^+(j)} x_e^{\hat{n}} \le w_j^{\hat{n}} y_{\hat{n},j}^{i,l} \qquad \forall j \in C(\hat{n},i) \text{ with } b_j^{\hat{n}} = 0, \forall i, \hat{n} \in I, i \ne \hat{n}$$

$$s_l^i \le (1 - y_{\hat{n},j}^{i,l}) b_l^i \qquad \forall l \in D(i,\hat{n}), \forall i, \hat{n} \in I, i \ne \hat{n}$$

$$x_e^i \le u_e^i \qquad \forall e \in E^i, \forall i \in I$$

$$x^i \ge 0 \qquad \forall i \in I$$

$$y_{\hat{n},j}^{i,l} \qquad \text{binary}, \ \forall l \in D(i,\hat{n}), \forall i, \hat{n} \in I, i \ne \hat{n}$$

$$s_j^i \ge 0 \qquad \forall j \in V^i \text{ with } b_j^i < 0, \forall i \in I$$