Spatial models under consideration

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Spatial models of interest

- Spatial autoregression analysis.(SAR)
- Spatial error model.(SEM)
- Spatial Durbin model.(SDM)

Bayesian disease mapping models

- Poisson-Gamma model for relative risk estimation
- CAR model
- space-time interaction

W is a spatial weight matrix

- It is really a zero 1 adjacency matrix
- This matrix determines which areas share borders
- This matrix is row normalized

Before normalization: W = adjacency matrix indicating which sectors share borders: W =

$$\begin{pmatrix} & S1 & S2 & S3 & S4 & S5 & S6 \\ S1 & 0 & 1 & 0 & 0 & 1 & 0 \\ S2 & 1 & 0 & 1 & 0 & 0 & 0 \\ S3 & 0 & 1 & 0 & 1 & 0 & 0 \\ S4 & 0 & 0 & 1 & 0 & 1 & 0 \\ S5 & 0 & 1 & 0 & 1 & 0 & 1 \\ S6 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

After row normalization: W =

$$\begin{pmatrix} & S1 & S2 & S3 & S4 & S5 & S6 \\ S1 & 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ S2 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ S3 & 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ S4 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ S5 & 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ S6 & 1/3 & 0 & 1/3 & 0 & 1/3 & 0 \end{pmatrix}$$

$$\frac{1}{1-\rho W} = W + \rho W + \rho^2 W^2 + \rho^3 W^3 + \cdots \tag{1}$$

Spatial autoregression

contextual impact

$$y_t = \rho W y_{t-1} + XB + e_t \tag{2}$$

$$y_t(I - \rho W) = XB + e_t \tag{3}$$

$$y_t = (I - \rho W)^{-1} XB + (I - \rho W)^{-1} e_t \tag{4}$$

where W=spatial weight matrix, which may be a row normalized adjacency matrix, ρ = spatial autocorrelation matrix.

Spatial error model

spatial heterogeneity

$$y_t = XB + (I - \rho W)^{-1} e_t \tag{5}$$

where W=spatial weight matrix, which may be a row normalized adjacency matrix, ρ = spatial autocorrelation matrix.

Spatial Durbin model for random effects

spatial lags at the explanatory variable and error process

$$Assume\ u = \gamma x + v \tag{6}$$

$$(I - \rho W)y_t = (I - \rho W)xB + \gamma x + u_t \tag{7}$$

$$y_t = \rho W y_{t-1} + (B + \gamma)x - \rho BWx + u_t \tag{8}$$

where W=spatial weight matrix, which may be a row normalized adjacency matrix, ρ = spatial autocorrelation matrix [3, 24-29].

Space-time models

- Bernardinelli et al. (1995)

Referrences

Lawson, A. B., Browne, W., and Rodeiro, C.L.V. (2009)*Disease Mapping with WinBUGS and MLwin* New York, NY: John Wiley and Sons, Inc., 128ff.

Lawson, A. B. (2008)*Bayesian Disease Mapping Hierarchical modeling in spatial epidemiology*. Boca Raton, Fla: CRC Press, ..

Le Sage, J. and Pace, R. K. (2009) *Introduction to Spatial Econometrics* Boca Raton, Fla: CRC Press, 23-29.